REDUCTION OF RAIN-CROSSPOLARIZATION BY ITS DEPENDENCE ON POLARIZATION DIRECTIONS

Lin-shan Lee
EDUTEL Communications and Development, Inc.
701 Welch Road, Suite 225, Palo Alto, California 94304

Abstract

Reduction of rain-induced crosspolarization is vital if system designers are to fully exploit frequency reuse. In this paper, a new method is developed for determining the expected crosspolarization level as a function of the polarization directions. It is also shown that we can reduce the crosspolarization by rotating the polarization angles between two fixed angles according to the wind directions.

Introduction

Due to an expected rapid growth in demand for communications capacity, substantial efforts are being made to reuse the same channel frequency with dual orthogonal polarizations. However, rain-induced crosspolarization severely limits the use of this technique, particularly in regions with frequent heavy rainfalls.

The rain-crosspolarization effect is due primarily to the nonspherical nature of the raindrops. The differential attenuation and differential phase shift of the drops were investigated in great detail by Oguchi [1-3], Morrison, Cross, Chu [4-6] and others. Theoretical analysis and experimental data of this effect were developed and measured, for example, by Saunders [7], Thomas [8], Watson, Arbabi, Eng [9, 10], Taur [11, 12], Chu [13], Semplak [14-17], Lin [18]. All of them consistently suggested the necessity of eliminating the crosspolarization if the frequency is to be reused by orthogonal polarizations.

In this paper we derive an expression for the level of crosspolarization as a function of the polarization directions, and then show how the crosspolarization levels can be reduced. The first step starts with a definition of the basic model for crosspolarization.

Basic Model

In analyzing the crosspolarization effect, Chu [13] chose the following model:

> Let the axis of symmetry of the oblate raindrop be oriented with respect to the horizontal direction at an angle θ, called the "canting" angle, and with respect to the propagation direction at angle γ. Let x_1, x_2 be the transmitted signals, y_1, y_2 be the received signals propagated through the raindrops in the horizontal and vertical directions. A typical situation is indicated in Fig. 1. If all raindrops are aligned in the same angle θ, and $\gamma = \pi/2$, then y_1, y_2 will be [13]

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} A' & \delta' \\ \delta' & B' \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{1}$$

where
$$A' = T_2\cos^2\theta + T_1\sin^2\theta \tag{2}$$
$$B' = T_1\cos^2\theta + T_2\sin^2\theta \tag{3}$$
$$\delta' = (T_2 - T_1)\sin\theta\cos\theta \tag{4}$$
$$T_1 = e^{-(\alpha_1 + j\beta_1)L} \tag{5}$$
$$T_2 = e^{-(\alpha_2 + j\beta_2)L} \tag{6}$$

and where α_1, α_2, β_1, β_2 are the attenuation and phase shift constants for the fields in the directions of the axes of the drops, and L is the length of the path.

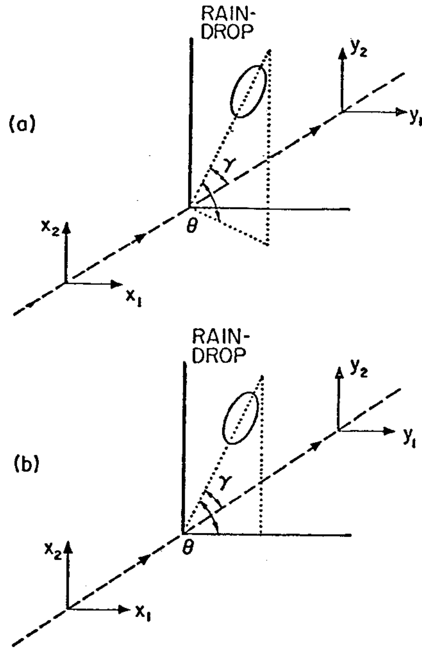

Fig. 1. The geometry in rain-induced crosspolarization effect.

(a) $\gamma \neq \frac{\pi}{2}$, *(b)* $\gamma = \frac{\pi}{2}$

Chu [13] also showed that the above model also holds, in general, even when $\gamma \neq \pi/2$ if T_1, T_2 are slightly modified to reflect the effect of γ.

However, in fact, the canting angle, θ, has a wide range of statistical distribution. Chu solved this problem [13] by introducing two parameters, ε and θ_{eff}, to take into account this variation. Both the parameters represent to some extent the "average effect" of the various canting angles and can be obtained by comparing the measured data and the calculated results. The final model is then

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} A & \varepsilon\delta \\ \varepsilon\delta & B \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{7}$$

where
$$A = T_2\cos^2\theta_{eff} + T_1\sin^2\theta_{eff} \tag{8}$$
$$B = T_1\cos^2\theta_{eff} + T_2\sin^2\theta_{eff} \tag{9}$$
$$\varepsilon\delta = \varepsilon(T_2 - T_1)\sin\theta_{eff}\cos\theta_{eff} \tag{10}$$

Chu [13] reported that the value of θ_{eff} and ε are two random variables with small variance. Most of the time they lie in the range

$$0.12 \leq |\varepsilon| \leq 0.18$$

$$20° \leq \theta_{eff} \leq 30°$$

and he estimated the median to be

$$|\varepsilon| \simeq 0.14$$

$$\theta_{eff} \simeq 25°$$

The sign of ε will depend on the wind component normal to the propagation path [13, 17, 19, 20].

Crosspolarization as a Function of Polarization Directions

Suppose we rotate the polarization directions of the signals x_1, x_2 both by an angle, ϕ, and let u, v be the actual transmitted signals in horizontal and vertical directions. Then,

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{11}$$

If u', v' represent the arriving horizontal and vertical signals in the receiving end, from Eq. (7),

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} A & \varepsilon\delta \\ \varepsilon\delta & B \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} \tag{12}$$

Suppose the polarizations of the receiving antenna are also rotated by the same angle, ϕ. The received signals y_1, y_2, will be

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} u' \\ v' \end{pmatrix} \tag{13}$$

These are shown in Fig. 2.

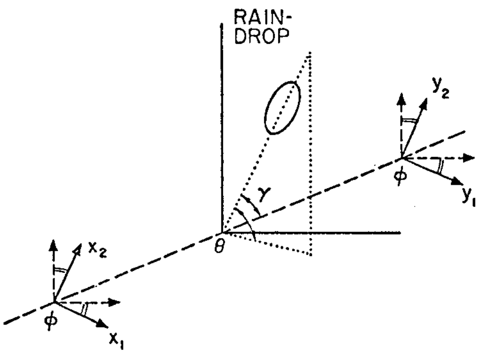

Fig. 2. The rotation of the polarization directions.

Combining Eqs. (11), (12), (13),

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A & \varepsilon\delta \\ \varepsilon\delta & B \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{14}$$

or

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{15}$$

where

$$T_{11} = A\cos^2\phi + B\sin^2\phi - \varepsilon\delta\sin(2\phi) \tag{16}$$

$$T_{22} = B\cos^2\phi + A\sin^2\phi + \varepsilon\delta\sin(2\phi) \tag{17}$$

$$T_{12}(\phi) = T_{21}(\phi) = \tfrac{1}{2}(A - B)\sin(2\phi) + \varepsilon\delta\cos(2\phi) \tag{18}$$

Equation (18) shows the crosspolarization as a function of the polarization directions when both the transmitting and receiving antennas are rotated by the same angle, ϕ. We can normalize Eq. (18) with respect to the original crosspolarization of the conventional horizontal and vertical scheme, $\varepsilon\delta$. Defining,

$$r(\phi) = \frac{|T_{12}(\phi)|^2}{|\varepsilon\delta|^2} \tag{19}$$

we have

$$r(\phi) = \frac{|T_{12}(\phi)|^2}{|\varepsilon\delta|^2} = \left[\left(\frac{A - B}{2\varepsilon\delta}\right)\sin(2\phi) + \cos(2\phi)\right]^2 \tag{20}$$

From Eqs. (8)-(10),

$$\frac{A - B}{2\varepsilon\delta} = \frac{1}{\varepsilon}\cot(2\theta_{eff}) \tag{21}$$

which is a real number; thus we can define

$$\frac{A - B}{2\varepsilon\delta} = -\cot(2\phi_0) \tag{22}$$

where

$$\phi_0 = -\tfrac{1}{2}\tan^{-1}[\varepsilon\tan(2\theta_{eff})] \tag{23}$$

The meaning of ϕ_0 will be clear very soon. Substituting Eq. (22) into Eq. (20), we can obtain a simple expression:

$$r(\phi) = \frac{\sin(2\phi_0 - 2\phi)^2}{\sin(2\phi_0)} \tag{24}$$

$r(\phi)$ is the normalized crosspolarization level, and is plotted in Fig. 3 for some values of ϕ_0. We find that $r(\phi)$ has actually a sinusoidal variation and a null point at $\phi = \phi_0$. Therefore, ϕ_0 indicates the optimal directions in which the orthogonal polarizations transmitted will not have any crosspolarized components and these components will be small, in the vicinity of ϕ_0. Note also that since the two signals are at right angles, $r(\phi)$ will repeat itself every 90°.

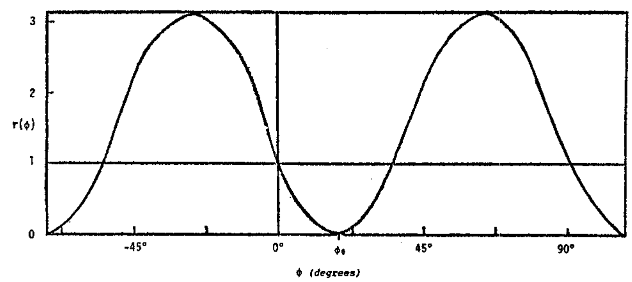

Fig. 3. The normalized crosspolarization level as a function of polarization directions.

Reduction of the Crosspolarization

Equation (23) shows that the optimal angle, ϕ_0, depends on ε and θ_{eff}, both of which are random. Hence ϕ_0 will also be random. Chu's [13] data shows that most of the time ε and θ_{eff} lie in the range,

$$0.12 \leq |\varepsilon| \leq 0.18$$

$$20° \leq \theta_{eff} \leq 30°$$

where ε will change sign according to the wind direction. We thus calculated the corresponding variation of ϕ_0, including the zero-crossing of ε, $|\varepsilon| \leq 0.12$. The result is plotted in Fig. 4. We find that ϕ_0 will also change sign according to the wind direction, just as ε does, and has a variation from about -9° to +9°. By Chu's median value, $|\varepsilon| = 0.14$ and $\theta_{eff} = 25°$, we have the median value of ϕ_0,

$$\phi_0\text{, median} = \pm 4.736° \quad \text{for} \quad \varepsilon = \pm 0.14$$

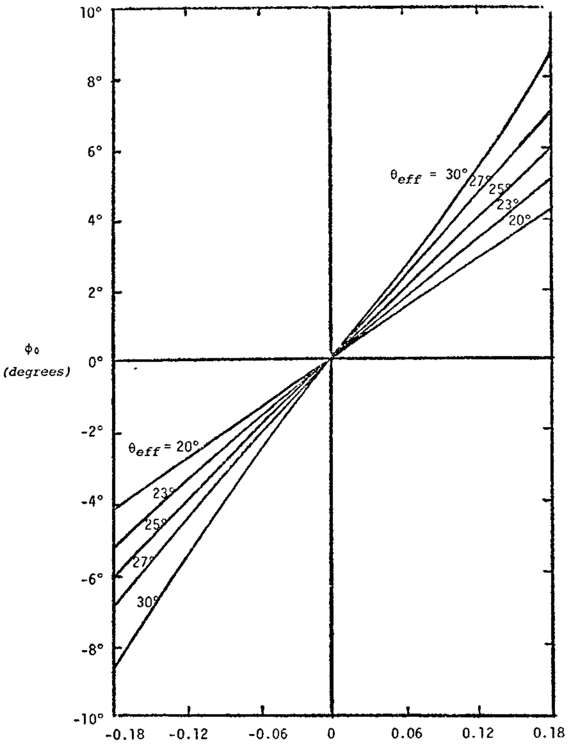

Fig. 4. The variation of the optimal direction, ϕ_0, with respect to the variation of ε and θ_{eff}.

Because of the random variation of ϕ_0, it will be difficult to set the polarization directions to the exact optimal values to obtain excellent isolation. However, it is possible to switch the polarizations between the two medians, ϕ_0, mean = ±4.736°, according to the wind direction. Defining,

$$S(\varepsilon, \theta_{eff}) = 10 \log \frac{|\varepsilon\delta(\varepsilon = 0.14, \theta_{eff} = 25°)|^2}{|T_{12}(\phi = -4.736°, \varepsilon, \theta_{eff})|^2} \tag{25}$$

We have the power ratio of the crosspolarization for the conventional horizontal and vertical scheme with positive median ε and θ_{eff} set to that for $\phi = -4.736°$, but with various ε and θ_{eff}. Positive value of S means a reduction of crosspolarization and negative S means an increase of crosspolarization compared to the median case. After simplification, it can be shown that

$$S(\varepsilon, \theta_{eff}) = 20 \log \cdot \left| \frac{\varepsilon_0 \sin(2\theta_{eff,0})}{\cos(2\theta_{eff})\sin(2\phi) + \varepsilon \sin(2\theta_{eff})\cos(2\phi)} \right| \tag{26}$$

where $\varepsilon_0 = 0.14$

$\theta_{eff,0} = 25°$

$\phi = -4.736°$

Equation (26) is plotted in Fig. 5. Attractively, Fig. 5 indicates that we can always have satisfactory reduction in crosspolarization so long as ε is positive. Similarly, if we do the same thing for $\phi = 4.736°$, we will find that we can have reduction so long as ε is negative. Therefore, we can reduce this cross-coupling simply by a rotation of the polarization (or the antenna) between two fixed angles according to the wind direction. However, if a wrong choice is made, the crosspolarization will be increased. This is also clear by inspection of Fig. 3.

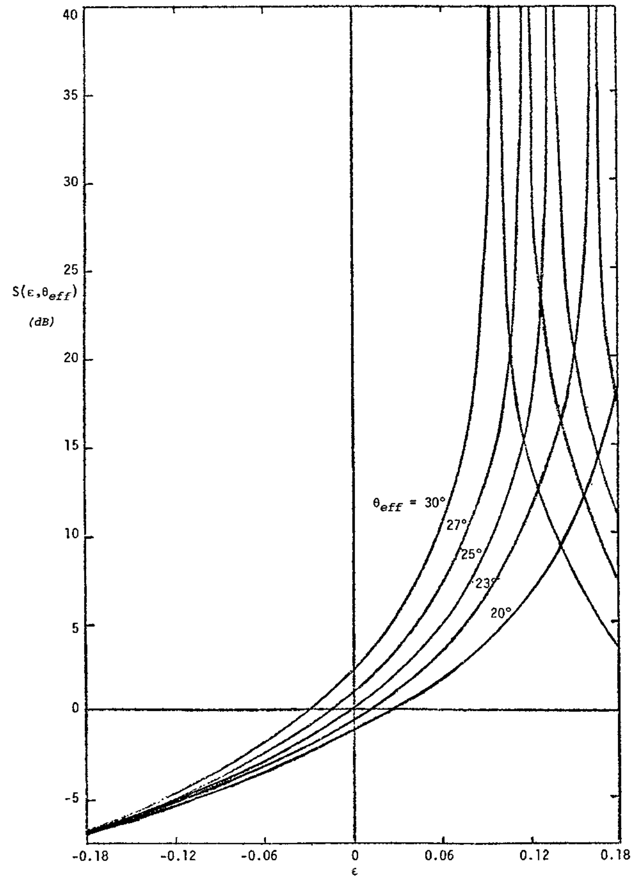

Fig. 5. Reduction and increase of crosspolarization compared to horizontal and vertical scheme when fixed, $\phi_0 = -4.736°$.

Conclusion

The rain-crosspolarization level is determined as a function of polarization directions. It is found from this function that we can reduce the crosspolarization by simply rotating the antennas between two fixed angles according to the wind directions. However, all the results are expressed in terms of the random parameters, ε and θ_{eff}. We would be able to make further improvement if more knowledge about the behavior of ϕ and θ_{eff} were available.

References

[1] T. Oguchi. "Attenuation of Electromagnetic Wave due to Rain with Distorted Raindrops (I)." J. of Radio Research Lab., Vol. 7, No. 33, Sept. 1960, pp. 467-483.

[2] T. Oguchi. "Attenuation of Electromagnetic Wave

due to Rain with Distorted Raindrops (II)." J. of Radio Research Lab., Vol. 11, No. 53, Jan. 1964, pp. 19-37.

[3] T. Oguchi. "Attenuation and Phase Rotation of Radiowaves due to Rain: Calculation at 19.3 and 34.8 GHz." Radio Science, Vol. 8, No. 1, Jan. 1973 pp. 31-38

[4] J.A. Morrison, M.J. Cross, T.S. Chu. "Rain-induced Differential Attenuation and Differential Phase Shift at Microwave Frequencies." Bell Syst. Tech. J., Vol. 52, No. 4, April 1973, pp. 599-604.

[5] J.A. Morrison, T.S. Chu. "Perturbation Calculations of Rain-induced Differential Attenuation and Differential Phase Shift at Microwave Frequencies." Bell Syst. Tech. J., Vol. 52, No. 10, Dec. 1973, pp. 1907-1913.

[6] J.A. Morrison, M.J. Cross. "Scattering of a Plane Electromagnetic Wave by Axisymmetric Raindrops." Bell Syst. Tech. J., Vol. 53, No. 6, July-Aug.1974, pp. 955-1019.

[7] M.J. Saunders. "Crosspolarization at 18 and 30 GHz due to Rain." IEEE Transac. on Antenna Propagation, Vol. AP-19, No. 2, March 1971, pp.273-277.

[8] D.T. Thomas. "Crosspolarization Distortion in Microwave Radio Transmission Due to Rain." Radio Science, Vol. 6, No. 10, Oct. 1971, pp. 833-839.

[9] P.A. Watson, M. Arbabi. "Rainfall Crosspolarization of Linearly and Circularly Polarized Waves at Microwave Frequencies." Electronic Letters, Vol.8, No. 11, 1 June 1972, pp. 283-285.

[10] P.A. Watson, C. Eng, M. Arbabi. "Rainfall Crosspolarization at Microwave Frequencies." Proceedings of IEE, Vol. 120, No. 4, April 1973, pp. 413-418.

[11] R.R. Taur. "Rain-depolarization: Theory and Experiment." COMSAT Tech. Review, Vol. 4, No. 1, Spring 1974, pp. 187-190.

[12] R.R. Taur. "Rain-depolarization: Measurements on a Satellite-Earth Propagation Path at 4 GHz." IEEE Transac. on Antennas and Propagation, Vol. AP-23, Nov. 1975, pp. 854-858.

[13] T.S. Chu. "Rain-induced Crosspolarization at Centimeter and Milimeter Wavelength." Bell Syst. Tech. J., Vol. 53, No. 8, Oct. 1974, pp. 1557-1539.

[14] R.A. Semplak. "Dual Frequency Measurements of Rain-induced Microwave Attenuation on a 2.6-kilometer Propagation Path." Bell Syst. Tech. J., Vol. 50, No. 8, Oct. 1971, pp. 2599-2606.

[15] R.A. Semplak. "The Effect of Rain on Circular Polarization at 18 GHz." Bell Syst. Tech. J., Vol. 52, No. 6, July-Aug. 1973, pp. 1029-1031.

[16] R.A. Semplak. "Simultaneous Measurements of Depolarization by Rain Using Linear and Circular Polarizations at 18 GHz." Bell Syst. Tech. J., Vol. 53, No. 2, Feb. 1974, pp. 400-454.

[17] R.A. Semplak. "Measurements of Rain-induced Polarization Rotation at 30.9 GHz." Radio Sci., Vol. 9, No. 4, April 1974, pp. 425-529.

[18] S.H. Lin. "An Occurrence of Very Heavy Rain on a 42-km Path." IEEE Transac. on Communication, Vol. COM-22, No. 5, May 1974, pp. 708-710.

[19] G. Brussaard. "Rain-induced Crosspolarization and Raindrop Canting." Electronic Letters, Vol. 10, No. 20, Oct. 1974, pp. 411-412.

[20] D.C. Hogg, T.S. Chu. "The Role of Rain in Satellite Communications." Proceedings, IEEE, Vol. 63, No. 9, Sept. 1975, pp. 1308-1331.

[21] Lin-shan Lee, James M. Janky. Corrective Measures for Propagation Effects: Rain Absorption, Rain Depolarization, and Antenna Sidelobes. Technical Report No. 9, Communication Satellite Planning Center, Radio Science Laboratory, E.E. Dept., Stanford University, Stanford, CA, 1976.